Diversity of the Tonal Structure of Chopin's *Etudes*

Miroslaw Majchrzak
Institute of Art, Polish Academy of Sciences in Warsaw, Poland

The original sound of Chopin's music results from his individual approach to harmony and tonality (See for example Tomaszewski[1]). Contemporary to Chopin, the term 'tonal unity', was understood as the absolute domination of the main key in a piece. A piece characterised by tonal unity had to be distinguished by an identical key in both its opening and its conclusion. In the late 1830s and early 1840s, 'off-key' treatment of this form appear in Chopin's output, and is characterised by a discrepancy between the initial and the ending key[2]. Certain deficiencies in the application of Riemann's method of harmonic analysis in studies on the Chopin harmonics have been noted by Ludwik Bronarski[3], author of "Harmonika Chopina" [The Chopin Harmonic]. Here, "side triads", often characterised by an element of significant independence, are crammed into functional inter-dependencies.

Andrzej Tuchowski[4] depicts a one-sided image of Frederic Chopin as a typical romanticist who constantly referred to emotional, rather than intellectual, aspects of creative work. This was a stereotypical view that, handed over from one generation to another, frequently influenced the perception of Chopin's forms. "This image, aggravated by suggestive and not infrequently exalted utterances of personages of the format of Schumann, Liszt, Noskowski, Paderewski, or Przybyszewski, made Chopin commentators' focus on the spectacular – that is, the strength of emotional influence, all the more that features such as structural cohesiveness of grand forms was popularly included in attributes of the effect of his production". On the contrary, Elzbieta Dziebowska[5] found that "Hundreds of pages were written on emotional content and expression of Chopin's music, the intellectual aspect of his output being neglected"; the same author wrote "analyses of rough drafts of Chopin's compositions indicate that the intellectual element played the primary part in [his] creative process." Attempts at applying new analytical methods to studies on Chopin's music appeared among Polish musicologists, such as the writings of Lissa[6] and Tuchowski[7]. Zofia Lissa[8] spoke of the need to perceive Chopin's composing technique through the prism of techniques characteristic to the music of later periods, including music of twentieth-century, with an emphasis on the intellectual elements to it.

This article attempts to compare the tonal structure of Chopin's *Etudes* with selected miniatures of other composers who were active in the same period, using an inventive and original research method, and the remarkable diversities in the way Chopin's pieces represented how this single genre is structured, will be demonstrated. I will present the diversity of the tonal structure of Chopin's *Etudes*: In Chopin's music a piece does not always start with the tonic chord. We also occasionally come across such a chord at the end of a given piece. I will try to answer whether or not the tonic key range always dominates Chopin's *Etudes*. Answering this question is important in light of the transformations of tonal music as it has evolved over centuries. This investigation will enable us to draw conclusions regarding the *Etudes'* historical position and their significance in terms of music written in later

periods, (such as the widely known introduction to Wagner's *Tristan und Isolde*, where chords related to the tonic are pushed to the background). The purpose of this study is to show the great diversity and originality of tonal structure in Chopin's *Etudes*, and will distinguish the various types of tonal structure of Chopin's *Etudes* through diagrammatic representation. The tonal structure of selected *Etudes* is reminiscent of the tonal structure of miniatures by composers of the former half of nineteenth-century, although this is not the case for every one of the works in question.

The research method applied herein enables a strict determination of quantitative relations between keys within the diatonic chords that have been identified[9]. The method of analysis enables the arrangement of a given set of keys to a hierarchical order, under which chords have been classified against the main key in which the piece is maintained. (In key range 2 – D major and B minor– the following chords will be classified, for example: DF#A, AC#EG, BDF#, AC#EGB, DF#AB, C#EG or DEF#GABC#)[10]. The method can serve musical analysis[11] as a tool by which to determine the main key of a piece, based upon various musical formats.[12] Alternatively, it can be helpful in discussions on the eternal dispute regarding the validity of two simultaneous modes: the major and the minor[13]. Keys are marked with consecutive integers, as illustrated in the table below:

Sign	Keys
…	…
9	D sharp major and B sharp minor
…	…
6	F sharp major and D sharp minor
5	B major and G sharp minor
…	…
2	D major and B minor
0	C major and A minor
…	…
-3	E flat major and C minor
-4	A flat major and F minor
…	…
-7	C flat major and A flat minor
…	…

Table 1. Keys

N.B.: (Minor keys appear in their natural form, hence the '2' key is a set of the following tones: D, E, F sharp, G, A, B, C sharp)[14].

Here, only diatonic chords have been subjected to this tonal structure analysis. Non-diatonic (N-D) chords are broken down into a separate line. Each diatonic chord has its own numeric value attached, which is calculated according to the arithmetical mean formula.

$$\textbf{Arithmetic mean} = x_1, x_2, x_3, \ldots, x_n / n$$

(where: $x_1, x_2, x_3, \ldots, x_n$ – keys wherein the tones of a given chord appear,
n – number of all keys.)

Example 1:
As a demonstration, let us, for example, take the G-B-D chord.
The tones of this chord appear in the following keys:
G – (-4, -3, -2, -1, 0, 1, 2)[15]
B – (0, 1, 2, 3, 4, 5, 6)
D – (-3, -2, -1, 0, 1, 2, 3)
The number of all keys where the G-B-D chord tones appear equals 21 (3 x 7)[16].

Arithmetic mean = 0,66...
(-4, -3, -2, -1, 0, 1, 2)+(0, 1, 2, 3, 4, 5, 6)+
+ (-3, -2, -1, 0, 1, 2, 3) / 7+7+7

Example 2:
Let us take any dyad, e.g. C-E.
The tones of this dyad appear in the following keys:
C – (-5, -4, -3, -2, -1, 0, 1)
E – (-1, 0, 1, 2, 3, 4, 5)
The number of all keys where the C-E chord tones appear equals 14 (2 x 7)[17].

Arithmetic mean = 0
(-5, -4, -3, -2, -1, 0, 1)+(-1, 0, 1, 2, 3, 4, 5)/7+7

The numerical value of the arithmetic mean of each diatonic chord will enable us to classify it within a given key range. The chord is classified within the range of a given key depending upon its arithmetic mean. The table below depicts selected key ranges:

Keys	Bb major G minor	C major A minor	G major E minor	A major F# minor
Key Range	KR -2	KR 0	KR 1	KR 3
from	-2,5	-0,5	0,5	2,5
to	-1,5	0,5	1,5	3,5

Table 2. Key Ranges

Example 3:
The arithmetic mean of F#-A-C chord is 1. It thus fits within KR 1, which is the key range for the keys of G major and E minor. Following this analysis, the dominance of these key ranges is observed. Metrical values are assigned to each chord, depending on the rhythmic value observed. Longer rhythmic values have, proportionally, larger metrical units than shorter ones. The arithmetic average for certain chords is on the borderline of two key ranges (divided by 2). (For example, the arithmetic average of the F-A-C-E chord is -0,5; the chord belongs to both KR -1 (F major and D minor) and KR 0 (C major and A minor).)

Basing our analysis upon the system discussed above, let us try and analyse a fragment of *Prelude in C Minor* by Frederic Chopin:

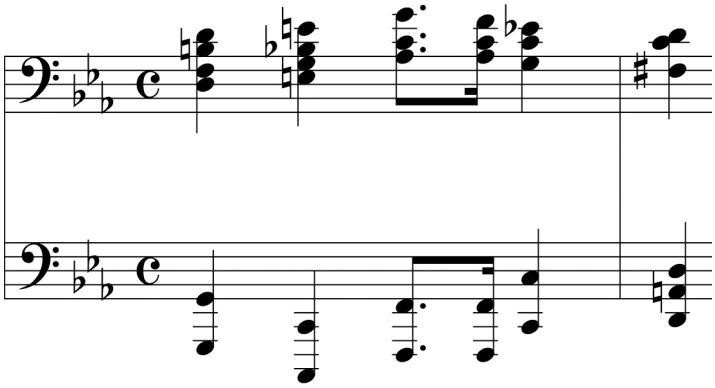

Figure 1. Example; Frederic Chopin, *Prelude in C Minor*

The data that can be taken from this is as follows:

Chords	Tones for all metrical Units	Key Range	Metrical Units
1	*B B B B* *F F F F* *D D D D* *G G G G*	KR 0	4
2	*Bflat Bflat Bflat Bflat* *G G G G* *E E E E* *C C C C*	KR -1	4
3	*G G G* *C C C C* *Aflat Aflat Aflat Aflat* *F F F F*	KR - 3	3
4		KR - 4	1
5	*Eflat Eflat Eflat Eflat* *G G G G* *C C C C*	KR - 3	4
6	*C C C C* *Fsharp Fsharp Fsharp Fsharp* *A A A A* *D D D D*	KR 1	4

Table 3. Metrical units and key ranges

In the above example, KR -3 (E flat major/C minor) prevails over all of the other key ranges. Neither one of the groups N-D nor U/P (unison/rests) can be found in the quoted fragment.

Before continuing to an analysis of the tonal structure of *Etudes* by Chopin, it is worth observing earlier pieces from the first half of the nineteenth-century. This analysis has been applied to a several miniatures by Robert Schumann and Felix Mendelssohn, as appropriate.

The graph below shows the tonal structure of *Chorale* from Schumann's *Album for the Young*.

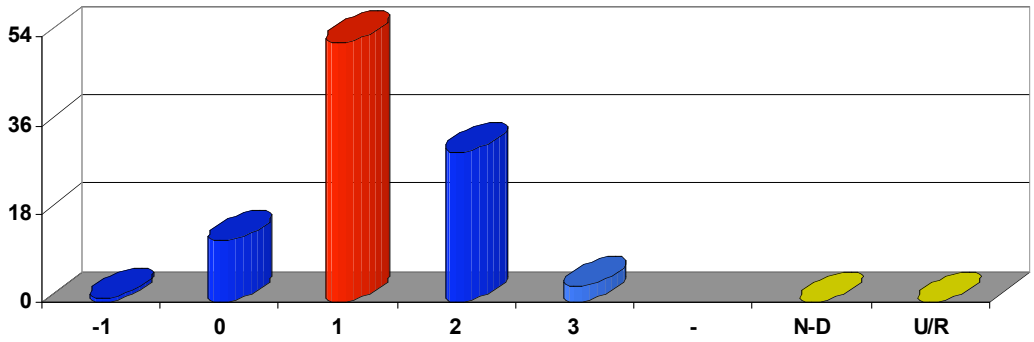

Figure 2. Robert Schumann, *Album for the Young*, *Chorale* (G major, KR 1)

The dominant key range is 1 (G Major and E Minor), as chords such as: G-B-D, D-F#-A-C, G-B-D-E, E-G-B have been classified under this particular key range. The G major tonic is one of the chords that have been classified in such a way. Here we can see that the tonic key range is predominant across those pieces. The same number (2) of key ranges can be found both to the right and to the left of the tonic range. The tonic key range is the (main) key range, in which we have classified the tonic chord. (In this example the tonic key range is KR 1: the next most frequent chords most likely to be found are in a circle of fifths from the tonic). To further describe the tonal structure of the piece under analysis, the tonic range is characterised by a very high number of appearances (a half of all the piece's chords can be classified as being in the keynote range). The range in question is situated symmetrically versus the key's other ranges, whose frequency of appearance gradually decreases as they recede from KR 1. No non-diatonic chords appear in this piece (N-D). This tonal structure proves characteristic to analysed pieces dating back to the Classical period[18]. Other works by Schumann have also a similar tonal structure.

For example:

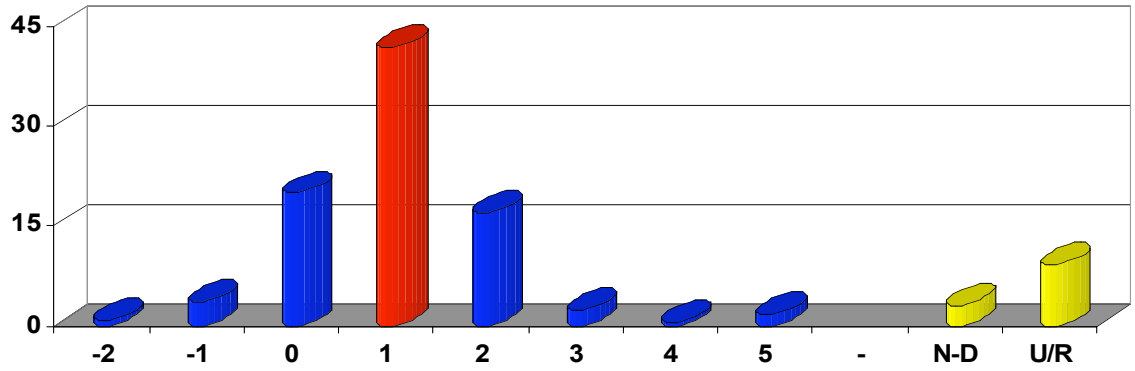

Figure 3. Robert Schumann, *Scenes from Childhood*, *About Strange Lands and People* (G major, KR 1)

In Mendelssohn's *Miniatures*, tonal structure diagrams are incredibly similar to those of Schumann, as the tonic range is predominant throughout. Again, a similar number of key ranges can be found to both the right (4) and left (3) of the tonic range. – i.e. the keynote range, as demonstrated:

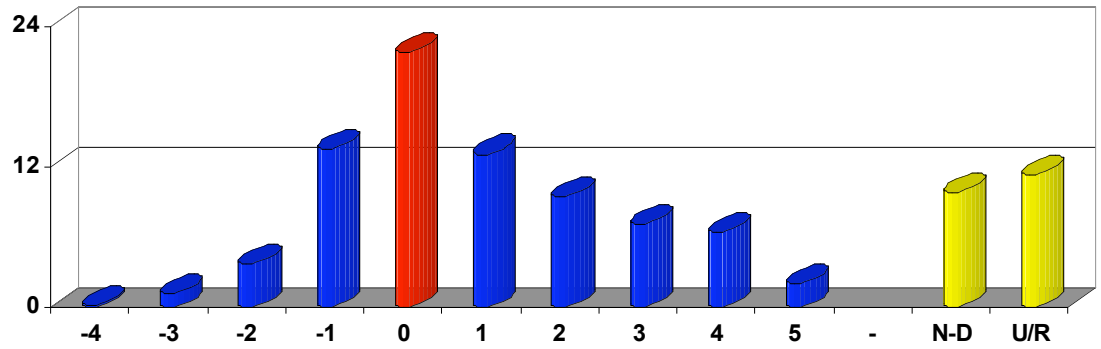

Figure 4. Felix Mendelssohn-Bartholdy, *Songs Without Words*, Op.19, No. 2 (A minor, KR 0)

However, the diagram above is not very representative of pieces written a minor key (See: Majchrzak 2008[19]). In this piece a dominant chord (E-G#-B) is classified within KR 4. The appearance of chords included in this key range is not frequent. In the key of C major, the tonic is included in KR 0, and the dominant, in KR 1: the chords thus appear in the neighbouring KRs. In the case of A minor, the minor tonic appears in KR 0, as the major dominant is classified under KR 4 (this being the E major chord, which may serve as the keynote for the E major key). In a traditional tonal structure the dominant key range (i.e. in C major – KR 1, in A minor key – KR 4) should have the 2nd or 3th range. In this example, the dominant key range (KR 4, Figure 4) has the 6th range. However, this irregularity is not very significant.

The early Romantic miniatures that have here been analysed, appear to be of a similar tonal structure: repeatedly the diagrams convey few key ranges, and frequent symmetry in their distribution. Dominance of the tonic key range (In F Major and D Minor – KR -1; in A Major and F sharp Minor – KR 3) is characteristic to all these pieces. However, we are confronted by an analogous situation in the case of Chopin's Songs[20].

In automatic musical analysis of tonal music we often confront numerous obstacles. MIDI (*Musical Instrument Digital Interface*), which is frequently used for computational analysis, does not distinguish any enharmonic "spelling". In other computational tonality analysis methods (for example: Chew[21], Krumhansl[22]) complications are encountered when modulation occurs. For the purpose of obtaining a tonal structure of Chopin's *Etudes*, I have used *Microsoft Excel* formulas (entering all musical material from a piece into each line). Diagrams of the tonal structure of Chopin's *Etudes* are based on C.F.Peter's *Musikverlag*. In this approach we can distinguish (vertical) non-diatonic material, which is also an important element of tonality. (For detailed remarks regarding tonality and chromatics, see Golab 1991[23]).

Chopin wrote twenty-seven *Etudes*, yet only few (between six and nine, depending upon the diagrammatic representation) have an unsophisticated tonal structure (similar to those pieces by Schumann or Mendelssohn, or pieces written in the Classical period as previously discussed). Let us use the *Etude in G flat major*, Op. 10, No. 5 as an example.

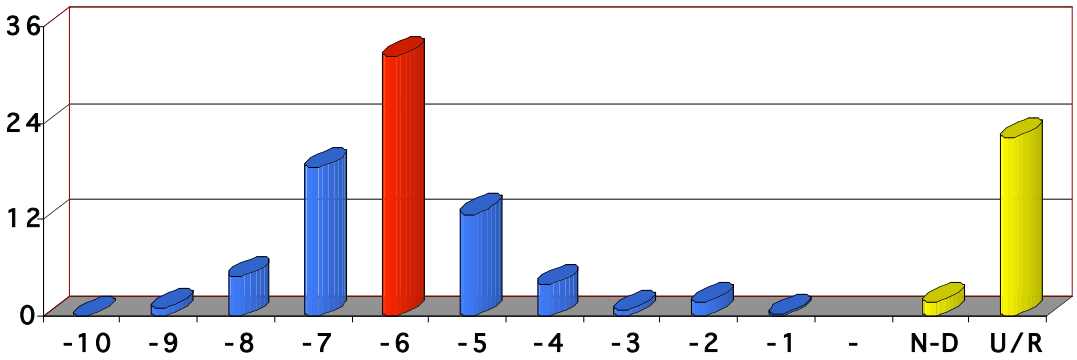

Figure 5. Frederic Chopin, *Etude in G flat major*, Op. 10, No. 5

The tonic range (KR-6) is clearly dominant in this piece. More than 30% of chords appearing in this piece fall within the key range G flat major/E flat minor. The piece's tonal structure is characterised by symmetry; there is one more key range on the right side of the key range (but it is worth noting that the frequency of its appearance is not considered).

Amongst Chopin's *Etudes* there are examples where the tonic range does not appear most frequently. No such cases appear in early-romantic miniatures or

classical pieces (Majchrzak 2005[24]). Let us take a closer look at *Etude in E flat minor*, Op. 10, No. 6:

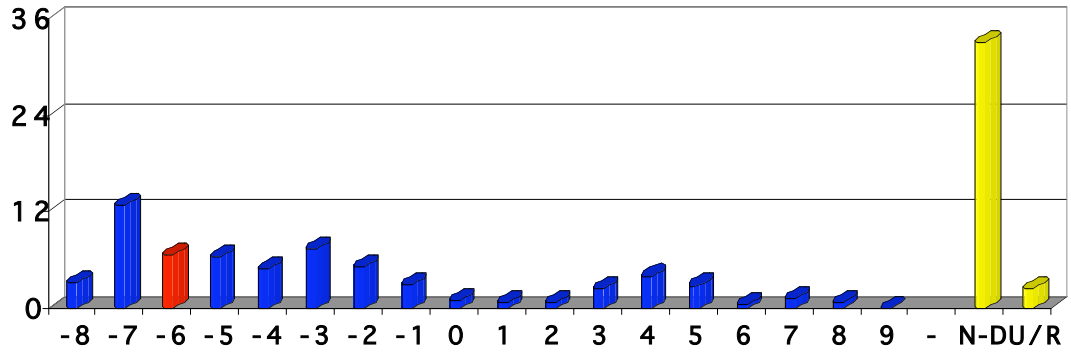

Figure 6. Frederic Chopin, *Etude in E flat minor*, Op. 10, No. 6

It is striking that in this particular piece non-diatonic chords appear quite so frequently, and the keynote range has not been allocated first "rank". Several tonal centres can be identified in the diagram: In comparison to the classical pieces, the *Etude in E flat major* has a highly asymmetrical distribution of key ranges. Left of the tonic range, we can only find two key ranges, with as many as fifteen of them appearing to the right. Such a situation seems quite natural, as it is written in the key of E flat minor. If we dealt with a piece written in a multi-sharp key, it would seem more natural to have a higher number of identifiable ranges left of the tonic range. However, Chopin does not always conform to this pattern.

Another *Etude* that has several tonal centres to it is *Etude in G sharp minor*, Op. 25, No. 6 – where the keynote range appears predominantly over other key ranges. Of a similar frequency of appearance to the tonic key range (KR 5), there is an evident group of non-diatonic chords (N-D, yellow).

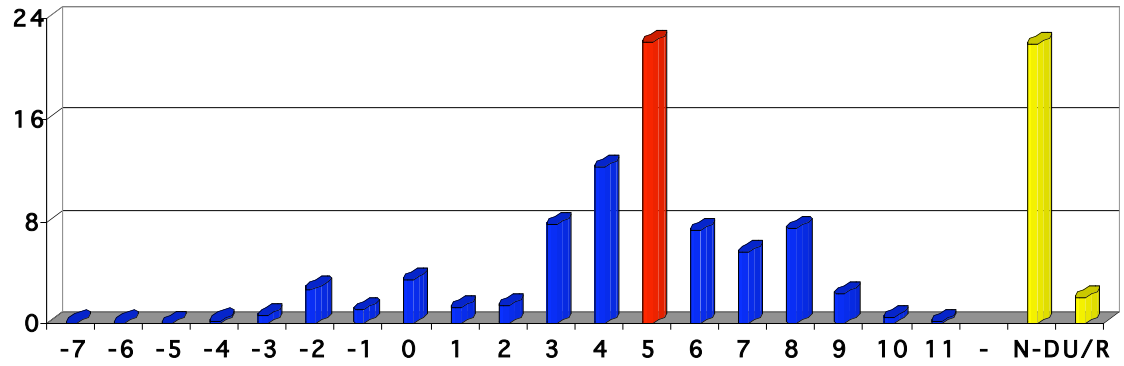

Figure 7. Frederic Chopin, *Etude in G sharp minor* Op. 25, No. 6

A further example is offered by pieces where "empty" ranges, or ranges with insignificant appearance, frequently appear between individual key ranges. Let us have a look at *Etude in D flat major*, Op. 25, No. 8:

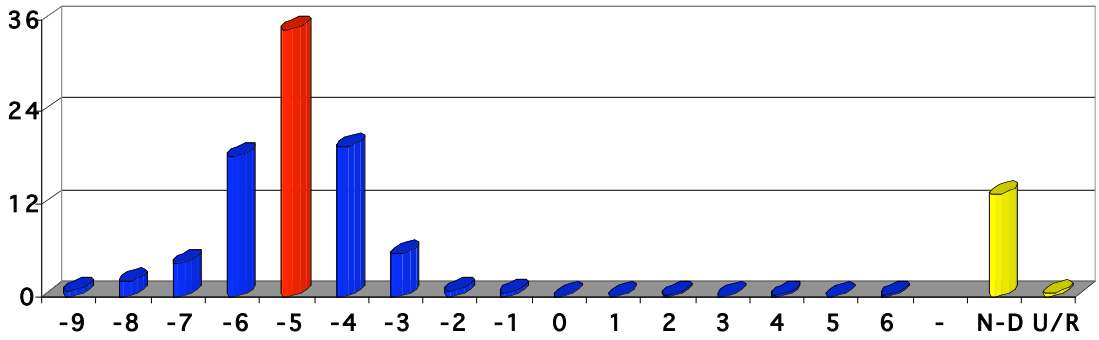

Figure 8. Frederic Chopin, *Etude in D flat major*, Op. 25, No. 8

The key ranges from KR -9 to KR-2 are situated closest to the keynote range, and form a typically Classical tonal structure. The composer would sometimes (yet very rarely indeed) use these chords classified here in key ranges, which are very remote from the tonic range, between which "empty" key ranges can be found. Despite a different number of key ranges appearing on both sides of the tonic range, the aggregate frequency of appearance in this piece of all the chords classified is on both sides of the tonic.

The previous section discussed the main types of tonal structure appearing in the Chopin's *Etudes*, but there are also a few variations to the models demonstrated. The presented examples have shown great originality in treatment of tonal structure in Chopin's *Etudes*, but the most diversity from a classical tonal structure can be seen in *Etudes* Op. 10 and Op. 25.

An entirely different situation can be seen in the last of Chopin's *Etudes*, composed during the latest period of his creative output. In contrast to the *Etudes* from collections Op. 10 and Op. 25, the *Etudes* without an opus number have a tonal structure that is far closer to that of his Classically structured pieces. Only in *Etude A flat major* do we meet other tonal centres (amongst which KR 0 and KR 4 with only a small number of appearances). *Etude in D flat major, No. 2* can be used as an example:

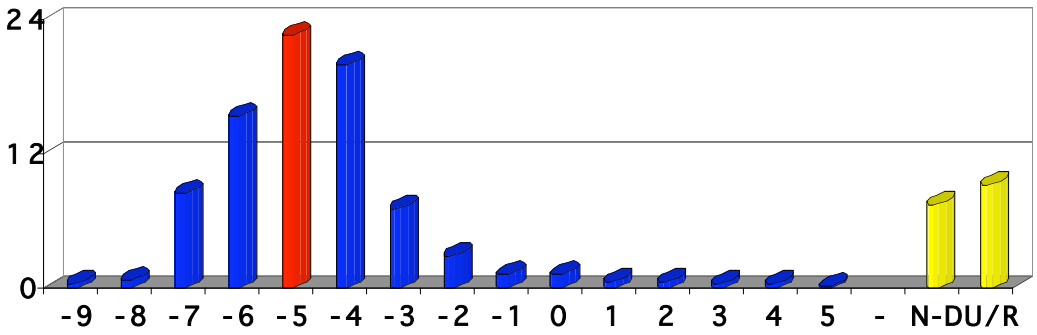

Figure 9. Frederic Chopin, *Nouvelle Etudes*, *Etude in D flat major*

Although the piece appears to contain a high number of key ranges, the frequency of appearance of extreme ranges is hardly significant at all. Despite the main key having as many as five flats, the tonal structure of the eight key ranges situated closest to the tonic range is significantly similar to the pieces that have a classical tonal structure. Only in the case of *Etude in A-flat major* can we find two small tonal centres. However, the frequency of appearance of key ranges is, again, insignificant.

The aforequoted examples of Chopin's *Etudes* have demonstrated his innovative treatment of tonal structure: it is not easy to find two *Etudes* with a very similar tonal structure. Wagner's *Tristan und Isolde* (1859) is deemed to mark a certain period in the development of music and tonality. The introduction is set in A minor, yet the composer avoided using the tonic triad. By applying this demonstrated original analytical method, the piece's main range (i.e. the A minor/C major key range, with its classified arrangements, e.g.: A-C-E; C-E-G; C-E; A-C-E-G) would definitely appear at less frequent a rate than the other key ranges. In Chopin *Etudes*, however, the main key range is often pushed to the background (especially when also taking into account the N-D group, or U/R). Already in Chopin's music (i.e. in the first half of nineteenth-century), we encounter certain features characteristic to the decline of tonality, whereas pieces by other composers contemporary to Chopin display a tonal structure still identifiable as Classical.

Acknowledgments. I am grateful to Rachel Foulds, Centre for Russian Music, Goldsmiths College, University of London, for help in the preparation of the final version of my paper.

References:

[1] Mieczyslaw Tomaszewski, *Chopin. Czlowiek, dzielo, rezonans* [Chopin: Man, Work, Resonance] (Poznan, 1998).

[2] Maciej Golab, *Chromatyka i tonalnosc w muzyce Chopina* [Chromaticism and Tonality in Chopin's Music] (Krakow, 1991).

[3] Ludwik Bronarski, *Harmonika Chopina [The Chopin Harmonic]* (Warszawa, 1935).

[4] Andrzej Tuchowski, *Integracja strukturalna w swietle przemian stylu Chopina [Structural Integration in the light of Chopin's Stylistic Change]* (Krakow, 1996).

[5] Elzbieta Dziebowska, *Chopin – romantyk, klasyk, modernista* [Chopin: Romanticist, Classicist, modernist] (Szkice o kulturze muzycznej XIX w., t. II, red. Zofia Chechlinska, Warszawa, 1973).

[6] Zofia Lissa, *Jednosc substancyjna kompozycji w swobodnych formach Chopina* (In Zofia Lissa, Studia nad tworczoscia Fryderyka Chopina [Studies in Chopin's music], Krakow: PWM, 1970).

[7] Andrzej Tuchowski, *Chopin's Integrative Technique and its Repercussions for 20th-Century Polish Music* (Polish Music Journal Vol. 2, Nos. 1-2., 1999).

[8] Zofia Lissa, *Harmonika Chopina z perspektywy dzwiekowej XX wieku [Chopin's Harmony from the point of view of the 20th century]* (In Zofia Lissa, Studia nad tworczoscia Fryderyka Chopina [Studies in Chopin's music], Krakow: PWM, 1970).

[9] This section is based on papers describing analytical method (For example: Majchrzak 2005, 2007).

[10] Miroslaw Majchrzak, *Irrelative System in Tonal Harmony* (Proceedings of First International Conference of Society for Mathematics and Computation in Music, 18-20 May 2007, Berlin, Germany; extended version in print).

[11] Mirosław Majchrzak, *Divergences and convergences of major and minor key distribution series in musical pieces of the tonal harmony supremacy period* (Master's Thesis, Wroclaw Academy of Music 2005, supervisor: Prof. Dr Stanislaw Krupowicz).

[12] Mirosław Majchrzak, *Analytic Capacities of an Original Tonality Analysis Method, Based on the Example of Chopin's Preludes op.28* (Proceedings of the 16th International Conference on Multimedia, 27–31 October 2008, Vancouver, BC, Canada, ACM New York, 2008).

[13] Miroslaw Majchrzak, *Mode-dependent Differences in Chord Classification under an Original Method of Tonal Structure Analysis* (Proceedings of 5th Sound and Music Computing Conference, 31st July – 3rd August 2008, Berlin, Germany, Universitätsverlag der Technischen Universität Berlin, 2008).

[14] The above set of tones can be arranged in a different sequence, producing the B minor scale in its natural variety: B, C sharp, D, E, F sharp, G, A.

[15] Keys: (A flat major and F minor), (E flat major and C minor), (B flat major and G minor), (F major and D minor), (C major and A minor), (G major and E minor), (D major and B minor).

[16] As for diatonic triads, one always deals with the value of 21, naturally.

[17] As for diatonic dyads, one always deals with the value of 14.

[18] Mirosław Majchrzak, *Divergences and convergences of major and minor key distribution series in musical pieces of the tonal harmony supremacy period* (Master Thesis, Wroclaw Academy of Music 2005, supervisor: Prof. Dr Stanislaw Krupowicz).

[19] Miroslaw Majchrzak, *Mode-dependent Differences in Chord Classification under an Original Method of Tonal Structure Analysis* (Proceedings of 5th Sound and Music Computing Conference, 31st July – 3rd August 2008, Berlin, Germany, Universitätsverlag der Technischen Universität Berlin, 2008).

[20] Mirosław Majchrzak, *Classicality of the Tonal Structure of Chopin's Songs* (Presentation at INTERNATIONAL CONFERENCE: Musical Analysis. Historia – theoria – praxis, Karol Lipinski Academy of Music in Wroclaw, 15th – 16th December 2008, in press).

[21] Elaine Chew, *Towards a Mathematical Model of Tonality* (Ph.D. dissertation. Operations Research Center, MIT. Cambridge, MA, 2000).

[22] Carol Krumhansl, *Cognitive Foundations of Musical Pitch* (New York: Oxford University Press, 1990).

[23] Maciej Golab, *Chromatyka i tonalnosc w muzyce Chopina* [Chromaticism and Tonality in Chopin's Music] (Krakow, 1991).

[24] Mirosław Majchrzak, *Divergences and convergences of major and minor key distribution series in musical pieces of the tonal harmony supremacy period* (Master's Thesis, Wroclaw Academy of Music 2005, supervisor: Prof. Dr Stanislaw Krupowicz).